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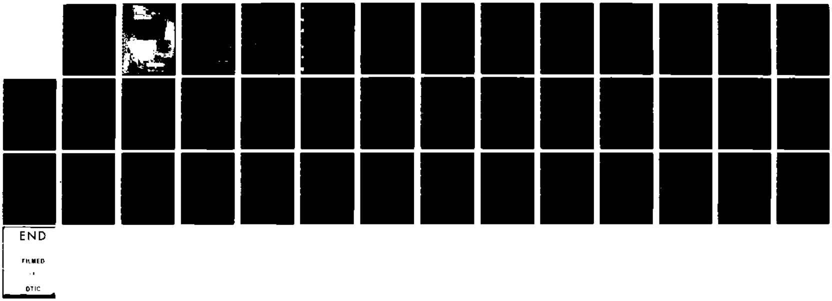

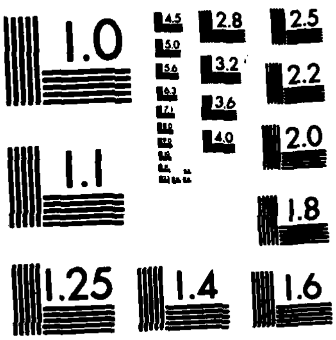
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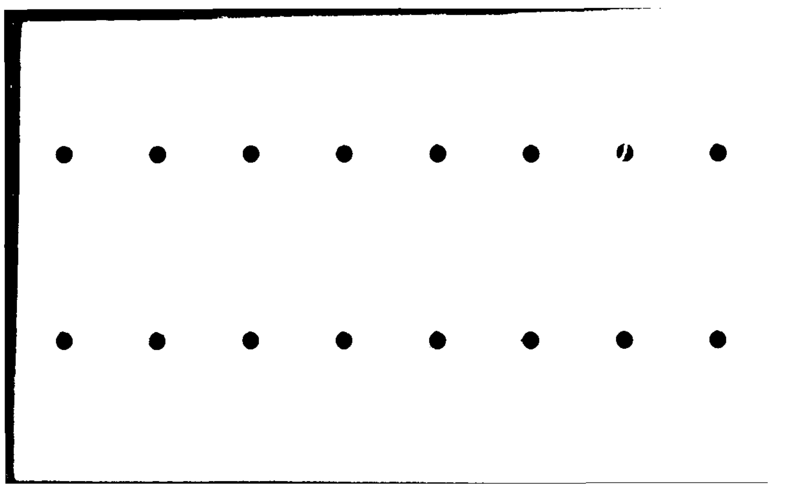

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A Study to Develop Improved EOQ Inventory Management Policies: Final Report

by

W. Steven Demmy

March 1982

WP-82-01
Decision Systems
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DOD LOGISTICS STUDY SUMMARY

1. LD NO. ~~[illegible]~~

2. DATE OF SUMMARY 02/83

3. TITLE AND ACRONYM A STUDY TO DEVELOP IMPROVED EOQ INVENTORY MANAGEMENT POLICIES: FINAL REPORT

4. STATUS	5. STARTING DATE	6. COMPLETION DATE	7. ORGANIZATION REPORT NO.
COMPLETED	06/81	03/82	WP-82-01

8. DLSIE SEARCH NO.	9. CONTRACT NO.	10. TYPE STUDY	11. TIME FRAME	12. COST
	F33600-80-C-0550	CONTRACT		35,000

13. SECURITY CLASS	14. DISTRIBUTION LIMITATION	15. MAN-YEAR EFFORT
UNCLAS	UNLIMITED	1 YR.

16. STUDY SPONSOR	17. PERFORMING ORGANIZATION
	DECISION SYSTEMS 2125 CRYSTAL MARIE DR. BEAVERCREEK OH 45431
RESPONSIBLE INDIVIDUAL	RESPONSIBLE INDIVIDUAL W. STEVEN DEMMY
AUTOVON	AUTOVON
COMMERCIAL NO.	COMMERCIAL NO. 1-513-426-8515

18. ABSTRACT (UNCLASSIFIED)

THIS PAPER SUMMARIZES THE RESULTS OF A STUDY TO DEVELOP IMPROVED DESCRIPTIONS OF EOQ ~~[illegible]~~ DEMAND AND LEAD TIME CHARACTERISTICS, AND TO DERIVE AND EVALUATE ALTERNATE REQUIREMENTS CALCULATIONS BASED UPON THIS NEW KNOWLEDGE.

19. CONCLUSIONS (UNCLASSIFIED)

20. RECOMMENDATIONS (UNCLASSIFIED)

21. IMPLEMENTING ACTIONS (UNCLASSIFIED)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A124547	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Study to Develop Improved EOQ Inventory Management Policies: Final Report		5. TYPE OF REPORT & PERIOD COVERED Final Report
		6. PERFORMING ORG. REPORT NUMBER WP-82-01
7. AUTHOR(s) W. Steven Demmy		8. CONTRACT OR GRANT NUMBER(s) F33600-80-C-0530
9. PERFORMING ORGANIZATION NAME AND ADDRESS Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Directorate of Management Science, Hq. AFLC/XRS, Wright-Patterson AFB, Ohio 45433		12. REPORT DATE March 1982
		13. NUMBER OF PAGES 32
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

EOQ, LEAD TIME DEMAND, INVENTORY MANAGEMENT

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Several recent studies have shown that some of the fundamental assumptions used to derive the requirements computation formulas currently used in the D062 Economic Order Quantity (EOQ) Buy Computation System provide a poor approximation to the actual characteristics of AFLC EOQ items. This paper summarizes the results of a study to develop improved descriptions of EOQ demand and lead time characteristics, and to derive and evaluate alternate requirements calculations based upon this new knowledge. Four new requrements computation formulas were identified that appear to provide significant cost-effectiveness improvements.

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EXECUTIVE SUMMARY

Overviews:

Several recent studies have shown that some of the fundamental assumptions used to derive the requirements computation formulas currently used in the D062 Economic Order Quantity (EOQ) Buy Computation System provide a poor approximation to the actual characteristics of AFLC EOQ items. This paper summarizes the results of a study to develop improved descriptions of EOQ demand and lead time characteristics, and to derive and evaluate alternate requirements calculations based upon this new knowledge.

APPROACH

Historical data was collected and analyzed discribing D062 item demand histories, requisition sizes and priorities, lead time variablility, and associated aircraft flying hour programs. We then developed five new requirements computations for detailed cost-effectiveness comparisons with the current D062 rules. Each of these new computations take different approaches in resolving the several statistical and analytic modeling issues associated with the development of a practical requirements methodology. The Inventory Systems Simulator (INSSIM) was then used to simulate how each of the proposed methods would have performed had the new method been used to manage D062 items during the CY73-79 interval. It was found that four of the five proposed policies provided significant cost-effectiveness improvements over the current D062 methods.

MAJOR FINDINGS

1. D062 demand patterns are much more erratic than is assumed in current D062 requirements formulas. This means that significantly higher safety stocks are needed to provide high levels of supply support than are permitted in the current computation.

2. Lead time variablility significantly increases the uncertainty associated with item lead time demands. However, the current D062 rules assume that lead time variability is neglegible.

3. Forecast errors for F-104 and F-5 aircraft items differ significantly from those associated with other items in the INSSIM Data Bank. We recommend that F-104 and F-5 items be separately analyzed in future INSSIM simulation studies.

4. Requisition count data contained in the D062 system appears to be very unreliable, and we believe that any calculation based on this data --including the current D062 safety level calculation-- is of little value. We believe that immediate actions should be taken to either (a) correct the data system problems associated with requisiton size statistics, or (b) implement alternate requirements formulas that do not require item requisition size statistics.

5. Lead time variability has major impacts upon requirements for safety stocks. However, accurate data describing lead time variability is extremely difficult to obtain from current AF data systems. Additional work to improve capabilities for lead time forecasting is greatly needed.

6. Four new computation policies were identified which appear to be significantly more cost-effective than the current D062 computations. However, because of the data problems described in (4) and (5) above, only one of these techniques is a candidate for immediate implementation.

RECOMMENDATIONS

1. We recommend that Policy Code 80, described in Appendix B of this report, be considered for immediate implementation in the D062 system. This computation policy appears to be significantly more cost-effective than the current D062 formulas, and it can be implemented using current D062 data. Further, only a few lines of computer code need to be changed to adopt these calculations. However, before implementation efforts are started, we recommend that EOQSIM be used to evaluate the transient effects of implementing this policy. Analytic calculations indicate that the new policy uses a significantly different strategy in setting safety stocks than the current D062 formulas. This means that, unless the phase-in is carefully managed, a large surge of buying activity could occur when the new formulas are first adopted. The EOQSIM projections should quantify the extent of this problem, and should provide a basis for testing alternate implementation strategies.

2. Estimates of D062 item lead times have a dramatic effect upon EOQ requirements computations, yet it is very difficult to evaluate the accuracy or variabilility of lead time forecasts using current AF data systems. To provide a basis for future improvements in requirements computations, we recommend that routine methods be developed for measuring the variability of item lead time and the accuracy of lead time forecasts, and for reporting this information within the D062 system.

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OVERVIEW

At present, AFLC Economic Order Quantity (EOQ) inventory management policies are based upon many of the commonly accepted assumptions described in inventory management texts, and simulation studies have shown that these methods are significantly more cost-effective than previously used policies. However, several recent studies have shown that several of these fundamental assumptions provide poor approximations to actual AFLC item characteristics. In this paper, we summarize the results of a study to develop improved understanding of the characteristics of AFLC EOQ items, and to develop and evaluate alternate inventory management policies which exploit this improved knowledge. Several policies were identified which appear to be more cost-effective than current AFLC D062 requirements computations.

STUDY OBJECTIVES:

This study had three major objectives:

1. To develop improved understandings and characteristics of Economic Order Quantity demand processes

2. To develop alternate requirements computation techniques using a more precise description of EOQ item characteristics than is employed in current methods.

3. To compare the cost-effectiveness of the newly developed procedures with current D062 policies.

APPROACH

To accomplish the above objectives, we began by performing a number of statistical analyses of actual EOQ demand histories contained in the INSSIM data bank. This data bank contains actual demand histories for approximately 40,000 D062 items managed by the Sacramento, Oklahoma City, and Warner Robins Air Logistics Centers (ALC). These histories cover the CY71-CY79 interval, a total of 38 quarters of demand data. A major objective of these statistical studies was to develop an empirical model of the distribution of forecast errors for these items. We also collected statistical data describing item requisition sizes, requisition priorities, and item lead time variability. This latter information was primarily obtained from previous AFLC study efforts.

Armed with a more precise description of the characteristics of EOQ items, we developed several alternate analytical models of EOQ demand processes, and we then derived the associated "optimum" requirements computation policies. Each of these models provide a more detailed description of actual EOQ item characteristics than are utilized in the current D062 requirements computation. However, each alternate model takes a different approach to the combination of statistical estimation and mathematical modeling issues involved in the development of a

practical requirements computation process.

In the final stage of this study, we performed detailed simulation experiments to compare the relative cost-effectiveness of the newly developed policies with the current D062 requirements computations. In performing these simulation studies we constructed four separate item samples of approximately five hundred items each from information in the INSSIM Data Bank. We then used the Inventory Systems Simulator (INSSIM) to evaluate each of the proposed policies. Basically, our simulation experiments evaluated how each of the proposed policies would have performed in managing each of these sets of items during the CY73-79 interval. From these computer runs, a large number of cost-effectiveness curves were constructed and analyzed. As a result, we identified four new policies which appear to be superior to the current D062 requirements computation rules. One of these appears to be particularly suitable for immediate implementation.

Finally, we documented the results of this study in a series of eight detailed working papers. These reports are listed in Appendix A. In this paper, we summarize the major results of the study effort.

With this background, let us now discuss the major findings and conclusions obtained from this study. For convenience, we have organized our discussion into three major sections; these correspond to the Statistical Analysis, Analytic Modeling, and Simulation aspects of the study effort.

STATISTICAL RESULTS

As noted above, we began our effort by performing a number of statistical studies to develop improved understanding of the characteristics of EOQ items and of the sources of forecasting errors associated with these items. We began our statistical efforts by collecting actual and predicted flying hour data for each of the twenty-three aircraft represented in the INSSIM data bank. We then computed an "Accuracy Ratio", the ratio of total flying hours predicted for a given twelve month interval to by the associated actual flying program for that interval. For example, as of 1 January 1973, a total of 230,800 flying hours were forecast for B-52 aircraft for the interval January through December 1973. In fact, a total of 203,703 hours were flown by the B52 during this interval. Hence, the accuracy ratio associated with this forecast equaled (230,800)/(203,703) = 1.133. That is, the January forecast exceeded the eventually observed program by 13.3%.

Similar calculations were performed for each forecast made during the CY73-CY79 interval, and for each of the aircraft represented in the INSSIM data bank. The results of these calculations are presented in Figure 1. We then performed Analysis of Variance (ANOVA) tests to determine if there was a statistically significant difference among aircraft or among time periods with respect to flying program forecast accuracy. After deleting observations asociated with very low levels of flying activity, we found no statistically significant difference in flying program forecast accuracy performance among weapons or across time frames. Consequently, it appears reasonable to pool these estimates to describe flying program forecast accuracy. This pooled distribution is presented in Figure 2. As shown in the figure, flying program accuracy ratios range from 76.3 percent to 181.2 percent of the forecasted values, and forecasts averaged 12 percent above the flying program that was eventually observed. Hence, we conclude that errors in flying program forecasts are a potentially significant source of forecasting errors in EOQ requirements projections.

Next, we performed a number of correlation and Analysis of Variance studies to develop an improved description of the relationship between forecast EOQ usage rates and the actual demands for these items. In our initial efforts, we asked if there were any significant differences in the magnitude of demand forecast errors across aircraft, across item demand rate categories,across Air Logistics Centers, or across time periods.

Figure I. Forecast Accuracy Ratios for CY73-Cy79.

WEAPON	CY73	CY74	CY75	CY76	CY77	CY78	CY79	AVERAGE	C of V
B52	1.133	1.227	1.097	1.067	1.013	1.001	1.005----	1.078	.077
B57	1.371	.926	1.281	1.212	1.038	1.120	1.379----	1.190	.144
C118	1.160	1.278	1.340	1.644	1.000	1.000	1.000----	1.203	.199
C121	1.285	1.349	1.224	1.789	1.517	1.146	1.000----	1.330	.195
C123	1.355	1.176	1.094	1.011	.995	.972	.998----	1.086	.128
C130	1.280	1.127	1.157	1.090	1.065	1.021	.991----	1.104	.087
C131	1.121	1.104	1.417	1.182	1.189	.951	1.006----	1.139	.132
C135	1.108	1.149	1.223	1.136	1.062	.980	.979----	1.091	.083
C141	1.073	1.308	1.058	.961	.975	1.024	.989----	1.055	.113
F4	1.088	1.077	1.144	1.159	1.056	1.076	1.001----	1.086	.049
F5	.981	.959	1.463	.917	.894	1.009	1.014----	1.034	.188
F100	1.116	.935	1.035	1.017	1.066	1.128	.937----	1.033	.075
F101	1.014	.924	.987	1.033	1.027	1.087	1.026----	1.014	.049
F102	1.099	.763	1.360	1.114	1.441	1.125	1.000----	1.129	.199
F104	1.483	.803	1.501	1.000	1.812	.986	1.029----	1.231	.299
F105	1.279	1.023	1.172	1.038	1.114	1.155	1.236----	1.145	.083
F106	1.154	1.052	1.045	.966	.948	.968	.923----	1.008	.080
F111	1.086	1.084	1.142	1.594	1.259	1.386	1.047----	1.228	.163
FB111	1.800	1.330	1.030	1.008	.983	1.038	.982----	1.167	.261
T29	1.112	1.145	1.000	1.000	1.000	1.000	1.000----	1.037	.061
T33	1.160	1.076	1.226	1.110	1.025	.978	1.004----	1.083	.082
T37	1.151	1.090	1.121	1.128	1.011	1.055	1.039----	1.085	.048
T38	1.125	1.106	1.054	1.089	1.039	1.040	1.037----	1.070	.034
T39	1.200	1.146	1.546	1.181	1.132	1.022	1.011----	1.177	.152
TF111	1.194	1.127	1.122	1.465	1.178	1.315	1.034----	1.205	.119
CT29	1.114	1.134	1.713	1.182	1.189	.951	1.006----	1.184	.211
------	-----	-----	-----	-----	-----	-----	-----		
AVERAGE	1.194	1.093	1.214	1.157	1.116	1.059	1.026		
C. OF V.	.140	.134	.155	.191	.180	.100	.087		

GRAND MEAN 1.123

*During these periods, actual flying hours were so low that tre accuracy ratios are not meaningful. To prevent these points from biasing our results upward, the true values were replaced by 1.000.

Figure 2.

Frequency Distribution of
Forecast Ratios, CY 73-79.

MEAN = 1.123 SUM = 204.340

Variance= 3.0874917E-02

Std. Dev = .1757126 Coef. of Var = .15650236

Minimum = .763 Maximum = 1.812

Frequency Distribution

CELL	UP LIM	CNT	
0	.50	0	I
1	.60	0	I
2	.70	0	I
3	.80	1	I*
4	.90	2	I**
5	1.00	27	I***************************
6	1.10	70	I**
7	1.20	45	I***
8	1.30	13	I*************
9	1.40	10	I**********
10	1.50	5	I*****
11	1.60	4	I****
12	1.70	1	I*
13	1.80	2	I**
14	1.90	2	I**
15	2.00	0	I
16	2.10	0	I

We found that, yes, significant differences exist. In particular, we found that forecasting errors associated with F-104 and F-5 aircraft and forecasts associated with the CY75-76 time interval were significantly different in magnitude from those associated with the other 21 aircraft in the INSSIM data bank or with other time periods. Subsequently, we found that the F-104 was phased out of the USAF inventory during the CY75-76 interval, but was later reintroduced as a major Foreign Military Sales aircraft. Very significant forecasting errors were associated with this phase-out/phase-in process. On the other hand, we found that the F-5 is the only aircraft in the INSSIM data bank that has a continuously increasing program. Unfortunately, there are very few F-5 items in the INSSIM data bank, so the construction of a single data set consisting of only F-5 items is not a practical analysis alternative. Consequently, it is recommended that in future simulation studies, F-104 and F-5 items should be separately analyzed from the other INSSIM aircraft. Further, it is recommended that C-5, F-15, F-16, or similar D062 items be added to the INSSIM Data Bank to permit future analysis of items with ascending programs.

With the above information, we again repeated the correlation and ANOVA studies with all F-104 and F-5 items deleted. Because of the large number of items in our data set, statistically significant differences were still observed. However, the differences across aircraft and across time periods were of small magnitude and appear to be of little practical

impact. Consequently, we believe that statistical estimates of forecast errors may be pooled across aircraft and time periods for the remaining INSSIM items. However, major differences in error performance still exist among high and low activity item classes. Consequently, separate empirical models were developed for high and low activity item classes.

Next, we performed several studies to identify the shape of EOQ demand rates forecast error distributions. We found that, in general, the demand processes for EOQ items are much more erratic than is assumed in the current D062 formulas. At present, EOQ demand rates are forecast by first estimating the average usage rate over the past two years, and then adjusting this value for anticipated changes in flying program activity. It is then assumed that actual demand will be normally distributed about this forecasted rate. That is,in D062 it is assumed that the actual demand will be distributed about the forecasted value according to the familiar "bell-shaped" curve. In contrast, we found that the distribution of actual quarterly demand about the forecasted value is a highly skewed distribution with significant levels of probability in the right-hand tail of the distribution. This means that significantly higher levels of safety stock are required to provide a given fill rate than would be required if the normal probability model was in fact true.

For example, Figure 3 compares the normal distribution with observed distributions of EOQ demand forecast errors. The four distributions shown describe the relationship of actual

C.D.F. OF ZI FOR 1971-72 BASE YEAR

FOR Z1-Z4 AND NORMAL FOR SMALC SMGC=1

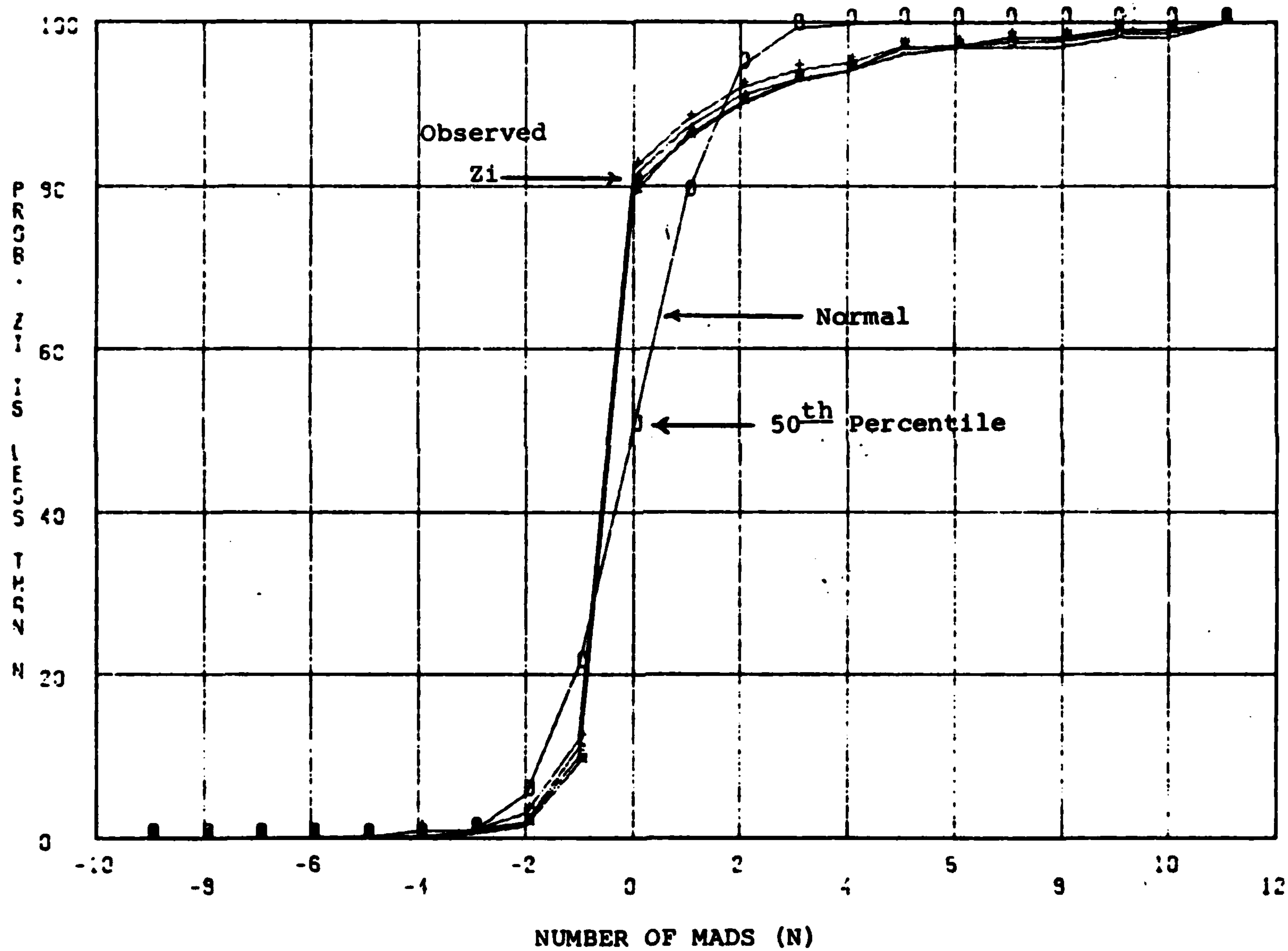


Figure 3. An Observed Distribution of Period Errors.

demands to the corresponding forecasted values for forecasts of one, two, three, and four periods into the future. The standardized error Zi is defined as

$$Z_i = \frac{\text{Actual Demand in Period } i - \text{Forecasted Demand for period } i}{\text{MAD}}$$

where MAD denotes the Mean Absolute Deviation of past demand, i.e. the average magnitude of past usage about the average value. Thus Z3 denotes the distribution of actual demnd in a quarter relative to the usage rate that was forecast three quarters earlier. As shown in the Figure, if demand is normally distributed there is a 50 % chance that actual demand will be less than or equal to the forecasted value, and it is almost certain that actual demand will be less than four MADs above the forecasted value. In contrast, we observed that for low demand items there is almost an 80 % chance that actual demands will be less than forecasted value, and there is at least a 5 % chance that demands for these items will exceed four MADs above the forecast. Almost identical curves are obtained for all time periods and across all Air Logistics Center item samples. For high activity items, the observed error distributions are closer to the normal curve, but the error distributions are still distinctly skewed. As noted above,the observed error distribution

implies that if high service levels are to be obtained, significantly higher safety stocks are required than is needed when forecast errors are norally distributed.

The above data describes the distribution of demand in a single period. We also studied the distribution of total net demand observed in a lead time one, two, . . . , or eight quarters relative to the D062 forecast usage for the combined iterval. We originally anticipated that the resulting distribution would be described by one of the standard probability models found in statistics and inventory management texts. However, we found that the observed lead time demand distribution does not fit any of these standard models. Consequently, we developed reasonable analytic approximations for the observed distribution of demand in a lead time. These approximations use exponential functions to approximate the observed lead time demand distribution. We refer to the resulting approximation as the exponential-constant, or EXP-CON model, since this model is simply an exponential approximation to the distribution of demand in a lead time that is a known constant. A comparison of the normal and EXP-CON models with the observed distribution of demand is presented in Figure 4.

Analysis of lead time variability data was another important research area. The above studies quantify the variability of a demand in a given number of periods. When replenishment lead times are also variable, the distribution of demand in a lead time is even more variable than is illustrated

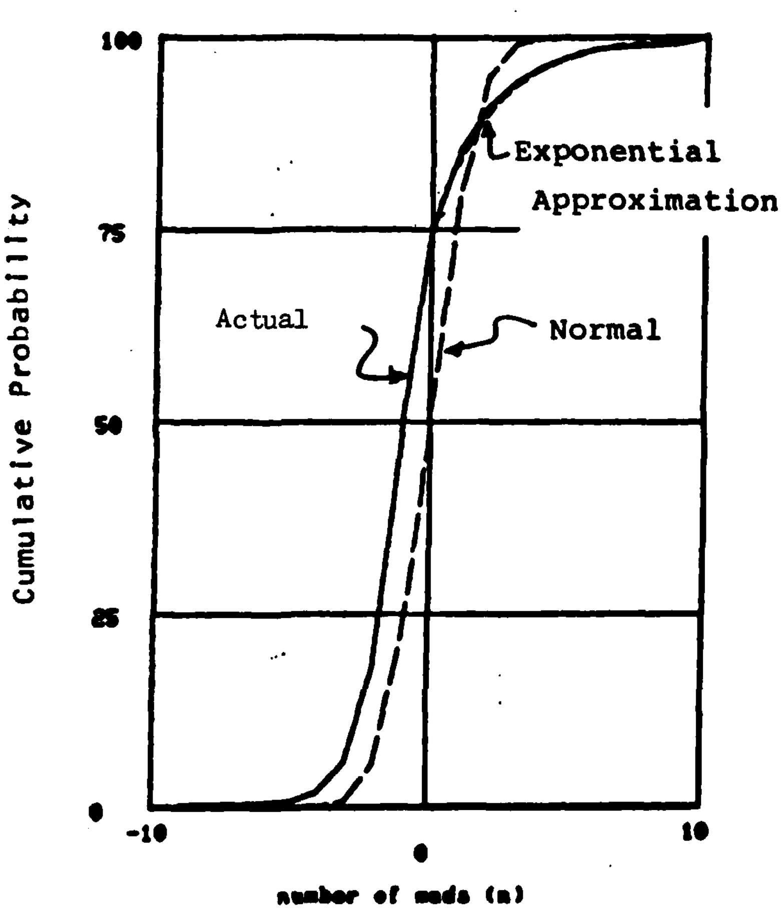


Figure 4. Comparison of the Normal and Exponential Models with Observed Demand Distribution.

in Figures 3 and 4.

We found that data describing lead time variability is extremely difficult to obtain. The best source of information which we could locate is contained in a 1981 study by Dr. Jack Hayya. Hayya studied the variability of replenishment lead times for 62 high activity D062 items and attempted to fit several probability distributions to the observed lead time data. Based on Hayya's results, we believe that the Gamma probability distribution provides the most reasonable single model of replenishment lead time variability.

The Gamma distribution is illustrated in Figure 5 using a coefficient of variation of .353. This is the median coefficient of variation value for the items studied by Hayya. That is, half of the items studied by Hayya had less variability than indicated in this figure, while approximately half had more variability. We used this model of lead time variability as the basis for our simulation studies. We believe that this is an adequate model for use in this study. However, more research is required if accurate lead time variability estimates are to be developed for all D062 items.

Finally, if we assume that demands in a given lead time are described by the exponential approximation illustrated in Figure 4, and if we also assume that replenishment lead time is independent of demand and described by a Gamma distribution such as that shown in Figure 5, then analytical methods may be used to compute the probability distribution of demand in a lead time.

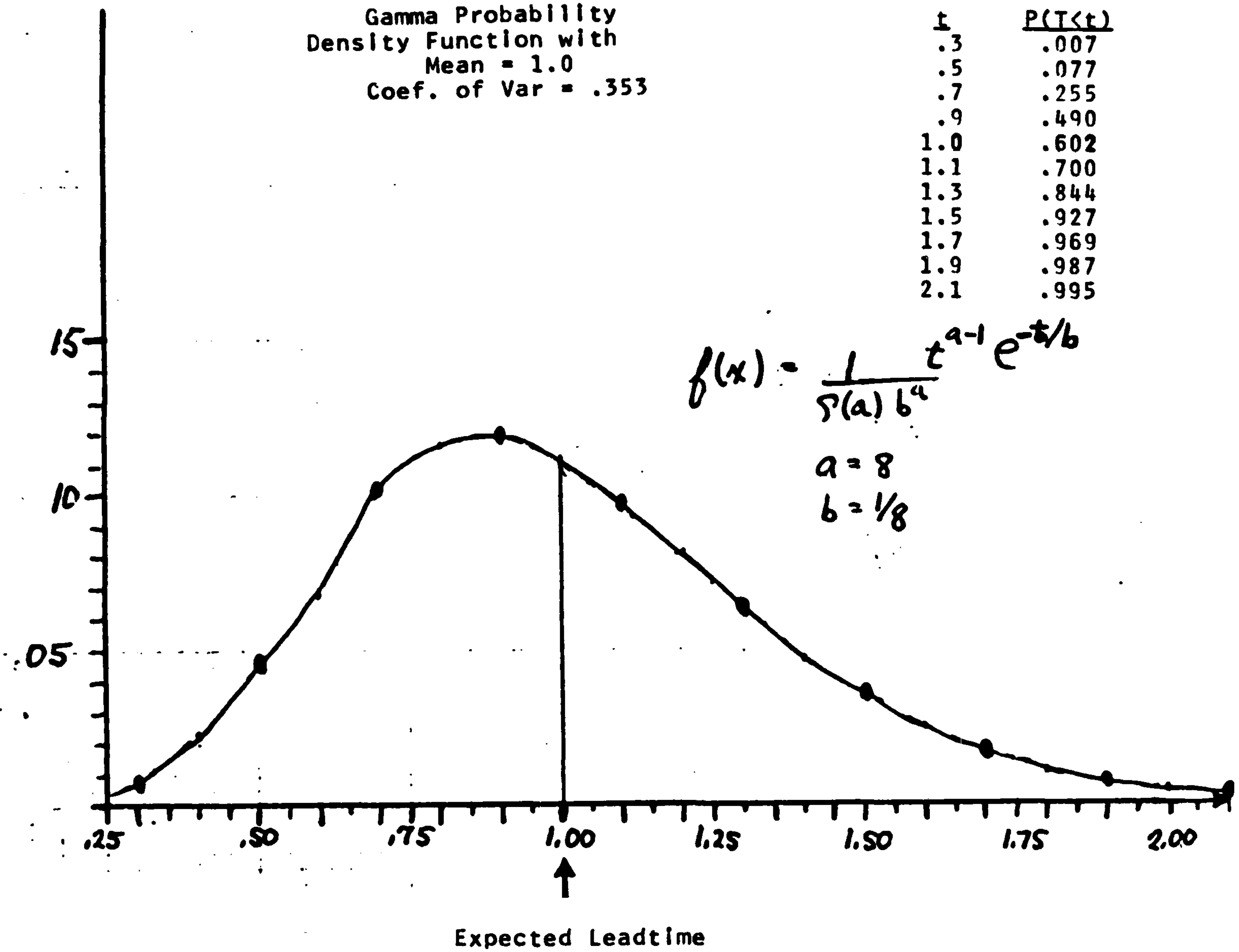


Figure 5. Gamma Density Function with Mean = 1.0 and Coefficient of Variantion = .353.

We refer to the resulting distribution as the Exponential-Gamma lead time demand model, or EXP-GAM for short.

The relationships among the Normal, EXP-CON, and EXP-GAM lead time demand models are illustrated in Figure 6. As shown on the figure, the EXP-GAM lead time demand distribution is even more erratic than the EXP-CON model. Thus, even higher safety stocks are required to provide high service levels in this latter case.

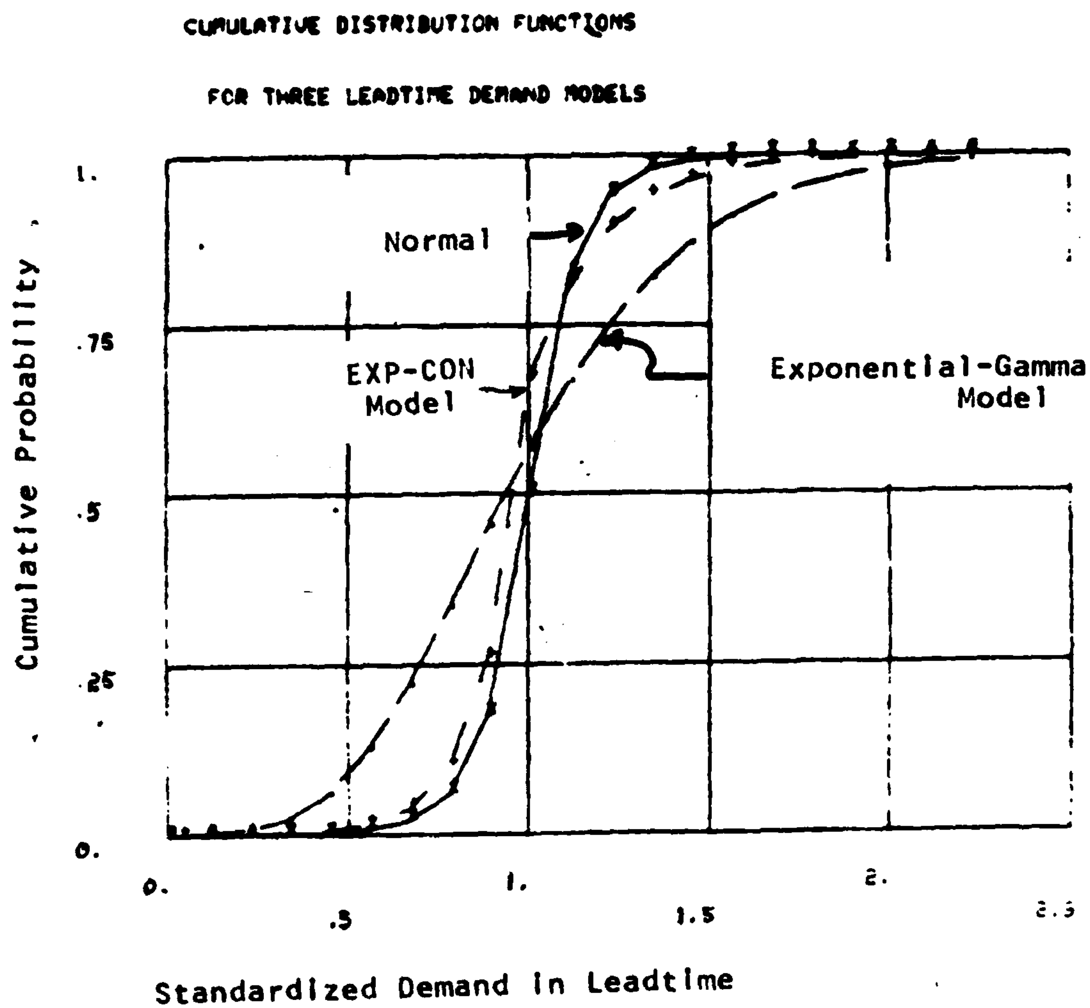


Figure 6. Laplace and Empirical Distribution for
Demand (Units/Qtr) = 3
Demand Coef. of Var. = .2
Leadtime Months = 9

ANALYTICAL MODELING

During the analytical modeling phase of this project we sought to develop new EOQ reorder point and order quantity calculations based upon a more accurate description of EOQ item characteristics than is employed in current D062 formulas. In this effort, we developed several alternate models of EOQ demand processes, and we then derived the associated "optimum" requirements computation policies. As noted above, each of these models provide a more detailed description of actual EOQ item characteristics than current formulas, but each of the alternate models resolve important statistical estimation and analytic modeling issues in different ways.

Basically, three categories of inventory management policies were developed for further testing. These are:

1. The current D062 policy and modifications of this policy to account for demand variability and for leadtime variability.

2. Policies derived from knowledge of the distribution of units demanded per requisition and of the distribution of item lead times.

3. Policies derived from the observed distribution of demand in a given lead time obtained from aggregate error statistics.

From these three basic categories, six policies were selected for detailed testing. These basic policies are described in Table I.

As shown in Table I, each policy was assigned a "Policy Code" to simplify discussions of the basic techniques. Policy Code 10 refers to the current D062 formulas. In studying individual item demand patterns, we observed that many item demand histories contain "spikes", i.e. very large erratic demands that are inconsistent with demands either before or after the spike. Hence, Policy Code 20 is identical to Policy 10, except that "spikes" are deleted from demand rate estimates to prevent an undesireable upward bias. Similarly, Policy Code 60 is also identical to the Policy 10 formulas, with the exception that the standard deviation estimate is modified to account for lead time variability.

In the second category of models, we began our analyis by considering a situation in which (a) customers arrive according to a Poisson process whose mean is proportional to flying program activity, (b) requisiton sizes obey a logarithmic distribution, (c) lead times are independent random variables which satisfy a gamma probability distribution, (d) all requisitions are assumed to be of equal priority, and (e) a continuous review (Q,R) order quantity, reorder point control system is being employed. With these assumptions, the distribution of demand in a lead time is described by the Logarithmic-Poisson-Gamma (LPG) distribution. This is a new probability distribution which was derived especially for this study.

Table 1

Inventory Management Policy Codes

Code	Inventory Management Policy
10	Current D062 Formulas
20	Current D062 Formulas, with outliers excluded from demand and variance estimates
60	Current D062 Formulas, with adjustments to standard deviation of lead time demand to account for lead time variability
70	Scaled Negative Binomial reorder point calculations
80	Constant Leadtime Exponential Forecast Error model
90	Exponential-Gamma Forecast Error Model

Unfortunately, the LPG distribution is quite complex, and significant computational difficulties are encountered in evaluating percentage points of this distribution for other than very low values of expected lead time demand. At the beginning of this study, we felt that the LPG distribution provided the most feasible detailed analytic description of AFLC EOQ demands. Because there are over 500,000 items managed by the D062 system, we considered the development of an efficient computational approach for the LPG distribution to be a primary research objective. Consequently, significant efforts were devoted to develop computationally efficient approximations to the LPG model. We found that a Scaled Negative Binomial probability distribution provides the required approximation. Details of the Scaled Negative Binaomial probability model are presented in Working Paper 81-04. On the other hand, computer programs for computing percentage ponts for the LPG and Scaled Negative Binomial distributions are presented in Working Paper 81-05. Policy Code 70 described in Table I corresponds to the use of the Scaled Negative Binomial model for requirements computations.

We know from sensitivity analyses of the LPG distribution that the distributon of individual requisition sizes has an important influence on the shape of the lead time demand distribution. Since the LPG model has four parameters, it is possible to more closely match the LPG model to the specific statistical estimates associated with a given item than is

possible using other methods. Unfortunately, this matching is not a free good. Because of the limited data available for any single item, single item estimates are subject to significant statistical estimation error. One method to reduce these statistical problems is to use aggregate item statistics. In the next paragraph, we discuss policies based on this latter approach.

The final category of analytical models which we studied utilize demand forecast error statistics aggregated over large populations of items to describe the distribution of demand in a given lead time. This is the Exponential-Constant, or EXP-CON, model discussed above. Although this model describes aggregate error performance, it is possible that no single item is accurately modeled by this distribution. However, by pooling the error statistics for a large number of items, we can obtain very precise statistical estimates of this aggregate error distribution. As shown in Table I, Policy Code 80 corresponds to the EOQ requirements computations that are obtained when the EXP-CON model is used to describe the distribution of lead time demand.

The final model selected for simulation evaluation is Policy Code 90. This policy corresponds to the Exponential-Gamma, or EXP-GAM, model of lead time demand discussed above. Recall that this model assumes that demand in a given lead time is discribed by an exponential approximation to the observed forecast error distribution, and a gamma distribution such as that shown in

Figure 5 is used to describe lead time variability. Numeric integration techniques are then used to compute the distribution of demand in a random lead time.

We also attempted to extend the above results to situations with two priority classes, but we encountered two major problems. First, we were unable to locate any useful source of individual item priority statistics. Hence, if a useful analytical model were developed, major data system changes would be required to support it. Second, the priority data which we were able to obtain indicates that fill rates for high priority requisitions were lower than for routine priority requisitions. This suggests that there is a complicated interdependence among stockage policy and the priority of submitted requisitions. Thus, models which assume independent priority streams appear inconsistent with the available data. Finally, even when independence is assumed, the mathematical analysis leads to expressions which are computationally impractical for use in the 500,000+ item D062 system. With sufficient effort, it may be possible to develop useful approximation equations for this case; however, we felt our efforts were better spent in improving the LPG and exponential approximation approaches.

SIMULATION RESULTS

The Inventory System Simulator (INSSIM) was used to evaluate each of the proposed rules under several different funding levels. INSSIM provides a detailed description of the D062 Economic Order Quantity Buy Computation System and uses actual Air Force demand histories to drive the simulation process. For this study, four item samples of appproximately 500 items each were selected from the INSSIM Data Bank. The samples SM.H and SM.L were selected from Sacramento Air Logistics Center records, while samples OC.H and OC.L were selected from Oklahoma City ALC records. The high activity samples SM.H and OC.H consisted of items which had net demands in CY71-72 which exceeded $5000 per year, while the samples SM.L and OC.L consisted of items with net CY71-72 demands which were less than $ 5000 per year.

Thirty-eight quarters of history covering the CY71-79 interval were available for each of these items. The first eight quarters of data were used to initialize the forecasting and inventory management rules, while the remaining 30 quarters of data were used to simulate the behavior of each of these rules. Thus, the simulation results evaluate how each of these rules would have performed had they been employed during the CY73-79 interval.

Each of the policies discussed above were simulated under several different funding levels, and our results were summarized in appropriate tables and cost-effectiveness plots. The detailed

results of the simulation effort are presented in Working Paper 81-07. Briefly, we found four policies whose simulation results provided significant cost-effectivenes improvements over the current D062 formulas. We found that Policy Codes 90, 80, and 70 consistently out-perform Policy Codes 60 and 10 for the requisition fill rate, unit fill rate, average unit delay, and average requisition delay measures. Policy Codes 90 and 80 performed particularly well for the low activity samples and for the unit-based measures of effectiveness. On the other hand, it was difficult to distinguish a superior policy for the requisition-based measures for the high activity samples.

When the long supply versus buy dollar curves were considered, we obtained mixed results. Policy Codes 90, 70, 60, and 10 were each ranked first or tied for first in at least one item sample, but Policy Code 80 was ranked last in all cases. This is a very interesting result, since Policy Code 80 performed very well with respect to each of the fill rate and average delay statistics reported above. Apparently Policy Code 80 achieves its improved supply-effectiveness versus buy dollar performance by taking higher risks that some of its safety stocks will later be classified as excess. However, Policy Code 80 has excellent support versus buy dollar curves on all the other measures of effectiveness studied.

IMPLICATIONS FOR THE MANAGEMENT OF REAL-WORLD INVENTORIES.

As discussed above, we obtained quantitative evaluations for each of the six policies in a simulated D062 environment. These results indicate that Policy Codes 90, 80, 70, and 60, in descending order of preference, are significantly more cost-effective than the current D062 rules. However, although the simulation provides a detailed description of the D062 system, there are several important differences between the D062 simulation model and the actual D062 environment. These differences must be considered in determining an appropriate policy for the management of the actual D062 system.

First, in our simulation model the mean and variance of procurement lead times were known with certainty, while in practice these parameters must be estimated from available data. We observed that several policies significantly out-perform the current D062 rules when accurate lead time data is available. Unfortunately, the required lead time data is not currently available in the D062 system, and it appears that a significant data processing effort would be required to routinely collect and update the needed information. Thus, implementation of Policies 90, 70, or 60 must either (a) await the development of such a system or (b) use regression or similar estimates as an interim measure. On the other hand, Policy 10 (the current D062 formulas) and Policy 80 do not require lead time variability parameters.

Second, in our simulation model we have perfect information concerning item requisition counts. In previous studies we have found that D062 requisition counts are extremely unreliable, and we believe that any formula that uses D062 requisition count data —including the current D062 formulas— is basing Air Force safety stocks on a random number generator. This problem appears particularly severe for the very large number of low activity items managed by the D062 system. Hence, we believe that the data processing rule of garbage-in, garbage-out would describe the results of implementing any rule that uses requisition count data from the current D062 system. Thus, rules 70, 60, and 10 could be expected to perform much worse in the "dirty-data" environment of the D062 system than they have performed in the simulated system.

Observe that Policy Code 80 is the only rule that avoids both of the severe data problems described above. Although Policy Code 90 provided results which were slightly superior to Policy 80, Policy 90 requires accurate estimates of lead time variability to deliver on its promise of superior performance. On the other hand, Policy 80 does not require these estimates. Policy 80 has another clear advantage. Mathematically, Policy 80 is even simpler than the current D062 formulas, and only a few lines of code would need to be changed to implement this rule in D062, in INSSIM, in EOQSIM, or in any other data system that uses the current D062 formulas. A flow chart documenting the logic and calculations for Policy Code 80 is presented in Appendix B.

On the other hand, Working Paper 81-02 documents the FORTRAN code needed to implement Policy 80 in INSSIM, and provides a logical framework for the development of computer codes for other effected systems.

In summary, we believe that significant improvements in inventory management effectiveness may be achieved by replacing the current D062 rules (Policy 10) by the Policy 80 rules as soon as possible.

Appendix A

Working Papers

Demmy, W. Steven, A Comparison of Forecasted and Actual Flying Programs for CY 1973-1979, Working Paper WP-80-06, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, October 1980, 53 pp.

Demmy, W. Steven, Plots of CY71-79 Demands and Returns for A Sample of Sacremento ALC D062 Items, Working Paper WP-81-01, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, May 1981, 132 pp.

Demmy, W. Steven, HEDGSIM ROUTINES for Leadtime Variability Inventory Policy Research, Working Paper WP-81-02, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, September 1981, 72 pp.

Demmy, W. Steven, A Statistical Analysis of the Distribution of D062 Demand in a Given Leadtime, Working Paper WP-81-03, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, September 1981.

Demmy, W. Steven, The Logarithmic Poisson Gamma Distribution: A Model for Leadtime Demand, Working Paper WP-81-04, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, September 1981, 15 pp.

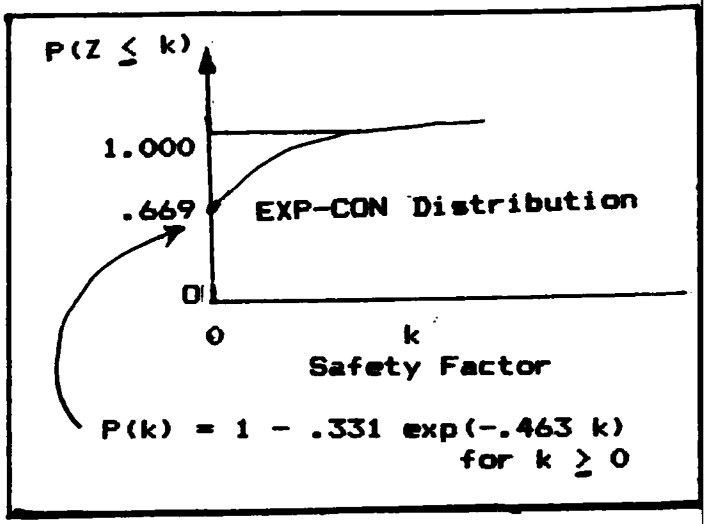
Demmy, W. Steven, A Sensitivity Analysis of The Logarthmic-Poisson Gamma Distribution, Working Paper WP-81-05, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, September 1981, 100 pp.

Demmy, W. Steven, A Comparison of Laplace Distribution with an Empirical Model of D062 Demand in a Leadtime, Working Paper WP-81-[illegible], Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, September 1981, 48 pp.

Demmy, W. Steven, An Evaluation of Alternate D062 Invenory Management Policies When Leadtimes are Random, Working Paper WP-81-07, Decision Systems, 2125 Crystal Marie Drive, Beavercreek, Ohio 45431, September 1981, 106 pp.

Appendix B

Policy 80 Flowchart



POLICY CODE 80

1. COMPUTE EOQ

$$Q = \sqrt{\frac{2AD}{IC}}$$

2. BOUND EOQ

IF $Q < .5D$, set $Q = .5D$
IF $Q > 3D$, set $Q = 3D$

A = Cost per order
D = Annual Demand Rate (Units)
I = Holding Cost (\$/\$-yr)
C = Item Unit Cost
= Shortage Cost per Unit Short
QMAD = Mean Absolute Deviation (MAD) of Demand per Quarter
LT = Lead Time in Quarters

3. COMPUTE OPTIMUM FILL RATE F,

$$F = 1 - QIC/(\pi\ D)$$

4. BOUND FILL RATE

IF $F < 0.$, set $F = 0$
IF $F > .9999$, set $F = .9999$

5. COMPUTE SAFETY FACTOR

Set $k = 0$

IF $F > .669$,
set $k = -[1/(.463)] \ln [(1-F)/(.331)]$

6. COMPUTE SAFETY LEVEL SL,

$$SL = k\ QMAD \sqrt{LT}$$

especially for this study.

END

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